Singular fate of the universe in modified theories of gravity

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ABSTRACT

In this Letter we study the final fate of the universe in modified theories of gravity. As compared with general relativistic formulations, in these scenarios the Friedmann equation has additional terms which are relevant for low density epochs. We analyze the sort of future singularities to be found under the usual assumption the expanding Universe is solely filled with a pressureless component. We report our results using two schemes: one concerned with the behavior of curvature scalars, and a more refined one linked to observers. Some examples with a very solid theoretical motivation and some others with a more phenomenological nature are used for illustration.

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1. Introduction

Refined astronomical observations of luminosity distances derived from Type Ia supernovae provide reliable evidence of the current cosmic speed up of the Universe (see [1] for the pioneering results and [2,3] for the latest). In fact, such measurements are the only direct indication of that phenomenon (see for e.g. [4]), but at the same time they are complementary with other key observations such as those of the CMB spectrum and the global matter distribution [5]. Explaining this surprising behavior in the large-scale evolution of the Universe represents a major theoretical problem in cosmology, and several approaches have been coined to try and provide a compelling answer to this riddle.

The main stream approach is to consider the Universe is filled with an exotic fluid, known as dark energy [6–8], but then one also has to demand the cosmic soup (made of dark energy and the rest of components) has some Goldilocks properties to comply with the observations. Alternatively, the idea that cosmic acceleration might be due to modifications to general relativity has received considerable attention as well (see [9,10] for reviews and for specific modifications). In such frameworks models displaying cosmic acceleration could be devised with less fine-tuning and unnaturality as compared to general relativistic dark energy scenarios [11]. Speculations in the direction of modified gravity are, in principle, legitimate as there are no cosmological tests probing scales as large as the Hubble radius. We only have reasonable evidence of the validity of the gravitational inverse square law up to 300 Mpc (through the ISW effect) [12]. However, the Hubble radius is two orders of magnitude larger, so our large-scale tests on general relativity are not stringent enough.

The additional degrees of freedom of these various settings, as compared to the standard picture of cosmology prior to the revolution ignited in 1998, have given rise to a collection of new cosmological evolutions with bold features, future singularities being the most perplexing ones. In this respect, attempts to classify somehow the sort of future singularities to be expected in new devised cosmological evolutions are of interest. A popular classification route in the literature [13] relies exclusively on properties of the curvature tensor and scalar quantities derived from it. From that perspective, a number of new terms in cosmology, such as the celebrated "big rip" [16], have been coined to designate extremality events associated with blow-ups of scalars constructed from the curvature tensor, along with less popular ones like "quiescent singularities" [17], "sudden singularities" [18], "big brake" [19] or "big freeze" [20] (the number of names is larger than the actual name of different extremality events).

Now, even though treatments of singularities in the fashion of [13] are of interest, there are subtle and most relevant properties inherent to cosmic evolution which can only be unveiled through the more sophisticated consideration of observers (see [14,15] for a detailed account). Indeed, curvature is a static concept, as it is only provides information of what happens at each event. Conversely, information retrieved from tracking the observers along their trajectories is more dynamical in nature, and therefore more enlightening if carefully analyzed. Interestingly, this scheme allows discussing whether the singularities encountered are weak or

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strong. Thus, if one's ultimate goal is to draw rigorous conclusions about the final fate in the Universe, both approaches are, in our view, complementary.

In this Letter we address the problem of future singularities in modified gravity cosmologies. We examine carefully the interrelation between the modifications and the singularities to be expected, and we try and give a unified vision by reporting our results using the scheme concerned with the behaviour of curvature scalars [13] and the one grounded on observers [14,15].

Ideally, modifications of general relativity should be derivable of a parent theory allowing for a covariant formulation of full-fledged field equations, otherwise, neither density perturbations nor solar system predictions could be computed. This is, actually, an aspect of the problem which does not affect our discussion, as we only work at the level of the Friedmann and energy conservation equations. Whenever the literature offers relevant examples for which the underlying theory is known, we will use them to illustrate our findings, but, occasionally, we will also resort to phenomenological examples.

The plan of the Letter is as follows. We propose a perturbative formulation of the Friedmann equations, for which two cases are distinguished depending on whether there is a critical energy density (which affects the form of the formulation). Then, we calculate the corresponding asymptotic expression of the scale factor, and building on earlier works we present our classification. We round up the dissertation with relevant examples and summarize in the last section.

2. Modifications of Friedmann equation

There have been many attempts to modify Einstein's theory of gravity from different points of view in order to cope with the observed acceleration of the expansion of the universe. One possibility arises from modifications to the Einstein–Hilbert action leading to the so-called $f(R)$ gravity theories (many aspect of this theoretical setup have been recently reviewed in [21]. The equations governing the large-scale geometry of the Universe in such settings are of fourth order in the metric approach, and, on top of that, for $f(R)$ gravity theories to evade compatibility issues with observational tests complicated models are required [22] (see however [23] for a different perspective). Mild applications of Ockham's razor principle, combined in graceful cases with physical motivations, have lead to the consideration that contending modified gravity schemes could perhaps be more advisable. This is the case of the proposals originated by assuming the Universe is a 3-brane embedded in a higher-dimensional bulk.

Instead of grounding our discussion in specific theoretical frameworks, we propose a perturbative expression for the Friedmann equation of an expanding universe, which intends to comprise most of the models in the literature.

With this aim in mind, we write a modified Friedmann equation in the form

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = h_0(\rho - \rho_*)^{\xi_0} + h_1(\rho - \rho_*)^{\xi_1} + \cdots. \tag{1}$$

Thus, we assume the squared Hubble factor can be expressed as a power series in the density ρ of the matter content of the universe around a specific value ρ_*, for which a qualitative change of behavior is expected. The exponents ξ_i are real and ordered, $\xi_0 < \xi_1 < \cdots$. The coefficient h_0 is obviously positive.

The equation system is closed by assuming, in addition, the validity of the usual energy conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{2}$$

The perturbative formulation represented by Eq. (1) can accommodate the Friedmann equations of the existing modified gravity

Table 1
Singularities in FLRW cosmological models.

η_0	η_1	η_2	Tipler	Królak	N.O.T.
$(-\infty, 0)$	(η_0, ∞)	(η_1, ∞)	Strong	Strong	I
0	$(0, 1)$	(η_1, ∞)	Weak	Strong	III
	1	$(1, 2)$	Weak	Weak	II
		$[2, \infty)$	Weak	Weak	IV
	$(1, 2)$	(η_1, ∞)	Weak	Weak	II
	$[2, \infty)$	(η_1, ∞)	Weak	Weak	IV
$(0, \infty)$	(η_0, ∞)	(η_1, ∞)	Strong	Strong	Crunch

proposals with a known parent covariant theory, as well as others with a phenomenological origin. Note, as well, that the Λ-cold dark matter (LCDM) or cosmic concordance scenario [24] is trivially comprised within this framework:

$$H^2 = h_0 + h_1(\rho - \rho_*),$$

with $\xi_0 = 0$, $\xi_1 = 1$, $h_1 = 8\pi G/3$, and $h_0 - h_1\rho_* = \Lambda/3$, so, actually, the parameter ρ_* is not fixed.

The main purpose of the modifications is to provide an accelerated evolution of the Universe without resorting to an exotic fluid, so it is usually assumed the Universe is simply filled with cold dark matter ($p = 0$), and this will be our working hypothesis as well. In this case, the energy conservation equation (2) can be straightforwardly integrated:

$$\frac{\dot{\rho}}{\rho} = -3\frac{\dot{a}}{a} \implies \rho a3 = K, \tag{3}$$

which gives a one-to-one map between the energy density and the scale factor through the integration constant K.

If we perform a power expansion of the scale factor in time,

$$a(t) = c_0|t - t_0|^{\eta_0} + c_1|t - t_1|^{\eta_1} + \cdots,$$

where the exponents η_i are real and ordered, $\eta_0 < \eta_1 < \cdots$, following [15] we shall be able to classify the singularities encountered at a time t_0. It is expected that most models allow this sort of expansion. However, there are models arising in loop quantum cosmology [25] which show accelerated oscillations that fall out of this scheme, though most of our conclusions can be extended to them.

The classification of singularities in weak and strong follows the ideas of Ellis and Schmidt [26]: The curvature may be finite or infinite at one event, but what is physically relevant is whether free-falling (or even accelerated) observers meet the singularity in finite proper time [27]. It is clear that if they take infinite time in reaching the curvature singularity, this would be indetectable.

Furthermore, if instead of ideal unextended observers we consider finite objects, the key issue is whether tidal forces at the singularity are *strong* enough to destroy them or *weak*, so that there could be objects that would survive beyond the singularity. This would mean that the weak singularity is by no means the end of the universe.

Following this ideas, Tipler [28] modelled extended objects by three perpendicular vorticity-free Jacobi fields travelling along a causal geodesic and forming and orthonormal frame with the velocity u of this. If the geodesic hits a singularity in finite time and the volume spanned by a set of three such vectors remains finite, the singularity is considered *weak*, since an object is not crushed. Tipler argues that in this case the metric could be generically extended beyond the singularity. Otherwise, if the volume is not finite for every set of vectors, the singularity is considered *strong*.

Thinking of cosmic censorship conjectures, Królak [29] suggested and alternative definition of strong curvature singularities that just required diminishing, instead of vanishing, volume of the finite object and is therefore easier to comply.

Compliance with these definitions for FLRW models can be checked resorting to integrals of the Ricci tensor along causal geodesics with respect to proper time τ [30]. If this integral diverges at a value τ_0

$$\int_0^\tau d\tau' \int_0^{\tau'} d\tau'' \, R_{ij} u^i u^j, \tag{4}$$

the causal geodesic meets a strong curvature singularity according to Tipler's definition.

And for Królak's definition divergence of this other integral at τ_0

$$\int_0^\tau d\tau' R_{ij} u^i u^j, \tag{5}$$

means meeting a strong curvature singularity.

The application of these results to FLRW models is summarized in Table 1, which is a simplified version of the one in [14].

The last columns refers to the classification of future singularities in [13]):

- Type I: "Big Rip": divergent a, ρ, p.
- Type II: "Sudden": finite a, ρ, divergent p.
- Type III: "Big Freeze": finite a, divergent ρ, p.
- Type IV: "Big Brake": finite a, ρ, p, but divergent higher derivatives.

Even though the modifications will only lead to acceleration for certain values of the parameter ξ_0, our forthcoming discussion on the asymptotic behavior of the scale factor is valid for any value of ξ_0, and it only relies on the ordering of the exponents. For this reason, our scheme comprises as well modifications to gravity which are not able to explaining the current acceleration, such as, for instance, the non-self-accelerating branch of the DGP modification scenario.

Inserting the modified Friedmann equation into the conservation equation one gets:

$$\frac{\dot{\rho}}{\rho} = -3\sqrt{h_0}(\rho - \rho_*)^{\xi_0/2} - \frac{3}{2}\frac{h_1}{\sqrt{h_0}}(\rho - \rho_*)^{\xi_1 - \xi_0/2} + \cdots. \tag{6}$$

Our purpose it to integrate the latter by considering all the possibilities which arise from different values of the parameters, and then use the aforementioned map between the energy density and the scale factor so that we can finally obtain asymptotic expressions for the expansionary behaviour of the models. Then, we will identify the specific late-time behaviour of the models, focusing on the existence of future singularities of various types. This classification resorts to earlier works by ourselves.

A separate treatment of the cases $\rho_* = 0$ and $\rho_* \neq 0$ cases is required, so we split the discussion into two subsections.

2.1. Absent critical density

In the case of a theory with no critical density, i.e. density $\rho_* = 0$, expressions get considerably simplified:

$$\frac{\dot{\rho}}{\rho} \simeq -3\sqrt{h_0}\rho^{\xi_0/2},$$

$$\rho(t) \simeq \begin{cases} \left\{\frac{3}{2}\xi_0\sqrt{h_0}(t-t_0)\right\}^{-2/\xi_0} & \text{for } \xi_0 \neq 0, \\ e^{-3\sqrt{h_0}(t-t_0)} & \text{for } \xi_0 = 0. \end{cases} \tag{7}$$

Correspondingly, in terms of the expansion factor we get

$$a(t) \simeq \sqrt[3]{K}\left\{\frac{3}{2}\xi_0\sqrt{h_0}(t-t_0)\right\}^{2/3\xi_0},$$

Table 2
Singularities in models without critical density.

ξ_0	Tipler	Królak	N.O.T.
$(-\infty, 0)$	Strong	Strong	I
$[0, \infty)$	Non-singular	Non-singular	Non-singular

which provides the following expected results:

- $\xi_0 < 0$. As matter density decreases smoothly, an eventual blow up of the corrections to the Friedmann equation is approached. At a finite time t_0 the scale factor becomes infinite, and the Universe experiences a type of singularity which has been called "big rip" [16] (type I in the classification in [13]).
- $\xi_0 > 0$. The matter density decreases and the scale factor increases smoothly as t grows towards infinity. This case comprises both quintessence-like behaviors for $\xi_0 \in (0, 2/3)$, and non-accelerated evolutions for $\xi_0 \geqslant 2/3$.
- $\xi_0 = 0$. The lowest order term is that of a cosmological constant, and we have to resort to the first correction with a positive exponent ξ_1, which leads again to an expression solvable as a Bernoulli equation:

$$\frac{\dot{\rho}}{\rho} \simeq -3\sqrt{h_0} - \frac{3}{2}\frac{h_1}{\sqrt{h_0}}\rho^{\xi_1},$$

$$\rho(t) \simeq \left\{e^{3\xi_1\sqrt{h_0}(t-t_0)} - \frac{1}{2}\frac{h_1}{h_0}\right\}^{-1/\xi_1}. \tag{8}$$

In this case

$$a(t) \simeq \sqrt[3]{K}\left\{e^{3\xi_1\sqrt{h_0}(t-t_0)} - \frac{1}{2}\frac{h_1}{h_0}\right\}^{1/3\xi_1}, \tag{9}$$

so, this situation represents an exponential expansion of the Universe, with a corresponding exponential decrease of matter density, with no future singularity at all.

Therefore, in the case $\rho_* = 0$, the modifications considered do not produce a qualitative change of behavior towards the future, except for dramatic modifications produced by negative exponents, which lead to a "big rip" singularity in the future.

We close this subsection with several examples which fit in $\rho_* = 0$ case of the general perturbative expression of H^2 we started from, namely they satisfy

$$H^2 = h_0\rho^{\xi_0} + h_1\rho^{\xi_1} + \cdots. \tag{10}$$

These results are summarized in Table 2.

The first case we consider for illustration is that of the power-law Cardassian models [31], for which

$$H^2 = \frac{8}{3}G\pi\rho\left(1 + \left(\frac{\rho}{\rho_{\text{card}}}\right)^{n-1}\right). \tag{11}$$

This expression can be accommodated into (10) with the following identifications between our parameters and those of the original reference: $\xi_0 = 1$, $\xi_1 = n < 2/3$, $h_0 = 8\pi G/3$, $h_1 = (8\pi G/3)\rho_{\text{card}}^{1-n} > 0$. The constant ρ_{card} signals the amount of matter energy density ρ below which the Cardassian corrections start to dominate ($\rho_{\text{card}} \sim \rho$).

DGP cosmologies [32–36] provide another interesting set of examples. If the brane has no tension and the bulk is the Minkowski spacetime, one has [34,37]

$$\frac{H}{H_0} = \sqrt{\Omega_{r_c} + \frac{8G\pi\rho}{3H_0^2}} \pm \sqrt{\Omega_{r_c}}. \tag{12}$$

Here H_0 is the value of the Hubble factor today, and Ω_{r_c} is the present value of the fractional energy density associated with the

scale at which the crossover to a corrections dominated regime occurs. In the perturbative formulation required for the discussion we get

$$H^2 = \Omega_{r_c}(1 \pm 1)^2 H_0^2 + \frac{8G\pi\rho}{3}(1 \pm 1) \mp \frac{16G^2\pi^2\rho^2}{9H_0^2\Omega_{r_c}} + \cdots. \qquad (13)$$

The self-accelerating branch [33,34] arises by taking the upper signs, and it is characterized by $\xi_0 = 0$ and $\xi_1 = 1$, whereas for the so-called normal branch [34,35], which arises by taking the lowers signs, one has $\xi_0 = 2$.

Finally, we can bring about the reinterpretation of Chaplygin-like cosmic evolutions as a modified gravity proposal [39,40]. In these frameworks one has

$$H^2 = \frac{8\pi G}{3}\left(A + \rho^{\gamma(\alpha+1)}\right)^{\frac{1}{\alpha+1}} \qquad (14)$$

with $\alpha > 0$ and $A = (3H_0^2/(8\pi G))^{(1+\alpha)}(1 - \Omega_m^{1+\alpha})$, and $\gamma = 1$ in the models considered in [39], whereas in the more general framework of [40] γ is free. Two cases are to be distinguished. In the $\gamma > 0$ case, the correspondence up to order $\gamma(1+\alpha)$ in ρ is given by $h_0 = (8G\pi/3)A^{1/(1+\alpha)}$, $\xi_0 = 0$, $h_1 = (8\pi G/(3(1+\alpha)))A^{-\alpha/(1+\alpha)}$, $\xi_1 = \gamma(\alpha+1)$. But in the $\gamma < 0$ the correspondence is rather different, as the identification up to order γ is given by $h_0 = 8G\pi/3$ and $\xi_0 = \gamma$.

In all the examples but the last one, according to the discussion above, no singular fate of the universe is faced. On the contrary, in the last kind of models the singularity is of "big rip" type.

2.2. Non-trivial critical density

New features appear for general modifications endowed with a non-trivial critical density ρ_*. For this case, we assume the matter density has an expansion around the critical value $\rho_* = \rho(t_0)$ at a time t_0:

$$\rho(t) = \rho_* + \rho_1(t_0 - t)^{\tilde{\eta}_1} + \rho_2(t_0 - t)^{\tilde{\eta}_2} + \cdots, \qquad (15)$$

and from the latter we obtain

$$a(t) = \frac{\sqrt[3]{K}}{\rho_*^{1/3}}\left(1 - \frac{\rho_1}{3\rho_*}(t_0 - t)^{\tilde{\eta}_1} + \cdots\right), \qquad (16)$$

so that the first exponents are the same in both expansions,

$$\eta_0 = 1, \qquad \eta_1 = \tilde{\eta}_1, \quad \ldots,$$

and we may drop the tildes.

At lowest order we have,

$$\frac{\dot{\rho}}{\rho} \simeq -3\sqrt{h_0}(\rho - \rho_*)^{\xi_0/2} = -3\sqrt{h_0}\{\rho_1(t_0 - t)^{\eta_1}\}^{\xi_0/2},$$

which upon the requirement of compatibility with Eq. (6) fixes the first exponent as

$$\eta_1 = \frac{2}{2 - \xi_0}. \qquad (17)$$

The following three cases are to be distinguished:

- $\xi_0 < 0$. Since $0 < \eta_1 < 1$, according to [15] or Table 1, these models have a singularity at t_0 with divergent H (a "big freeze" or singularity type III [13]), which is a weak curvature singularity according to Tipler [28], but strong according to Królak [29].
- $\xi_0 \in (0, 2)$. In this case $\eta_1 > 1$, so these models could show a weak singularity at t_0 according to [15] or Table 1 (sudden singularity [18] or type II in [13], or even type IV if $\eta_1 \geqslant 2$, $\xi_0 \geqslant 1$).

Table 3
Singularities in models with critical density.

ξ_0	ξ_1	Tipler	Królak	N.O.T.
$(-\infty, 0)$	(ξ_0, ∞)	Weak	Strong	III
0	$(0, 1)$	Weak	Weak	II
	$[1, \infty)$	Weak	Weak	IV
$(0, 1)$	(ξ_0, ∞)	Weak	Weak	II
$[1, 2)$	(ξ_0, ∞)	Weak	Weak	IV

- $\xi_0 = 0$. The cosmological constant term is dominant against modifications of the Friedmann equation. At first order, we have

$$\frac{\dot{\rho}}{\rho} \simeq -\frac{\eta_1\rho_1}{\rho_*}(t_0 - t)^{\eta_1 - 1} = -3\sqrt{h_0},$$

that is, we find a linear behavior for matter density:

$$\eta_1 = 1, \quad \rho_1 = 3\rho_*\sqrt{h_0}.$$

This being so, it turns out we have to expand the equation a bit further in order to reveal new qualitative behavior:

$$\begin{aligned}\frac{\dot{\rho}}{\rho} &\simeq -3\sqrt{h_0} - \eta_2\frac{\rho_2}{\rho_*}\sqrt{h_0}(t_0 - t)^{\eta_2 - 1} + \cdots \\ &= -3\sqrt{h_0} - \frac{3}{2}\frac{h_1}{\sqrt{h_0}}\{3\sqrt{h_0}\rho_*(t_0 - t)\}^{\xi_1} + \cdots \\ &= -3\sqrt{h_0} - \frac{3}{2}\frac{h_1}{\sqrt{h_0}}(\rho - \rho_*)^{\xi_1} + \cdots.\end{aligned}$$

Necessarily,

$$\eta_2 = 1 + \xi_1, \qquad \rho_2 = \frac{1}{2\eta_2}\frac{h_1}{h_0}\left(3\sqrt{h_0}\rho_*\right)^{\eta_2}, \qquad (18)$$

and therefore, according to [15] or Table 1,

$$a(t) = \frac{\sqrt[3]{K}}{\rho_*^{1/3}}\left\{1 - \frac{\rho_1}{3\rho_*}(t_0 - t) - \frac{\rho_2}{3\rho_*}(t_0 - t)^{\eta_2} + \cdots\right\}, \qquad (19)$$

there is a singularity at t_0 due to the lack of smoothness of the density and the scale factor. But this singularity is weak in both Tipler's [28] and Królak' [29] classification, so it does not exert any infinite distortion on finite objects going through it and cannot, therefore, be considered as a final stage of the Universe. It is a sudden singularity or type II in [13]) for which the scale factor and the density remain finite, but $\dot{H}$ blows up. It is worthwhile mentioning that milder singularities for which H and also $\dot{H}$ are finite (type IV in [13]) could, in principle, appear within this framework, but they would involve choosing $\eta_2 \geqslant 2$, and thereby $\xi_1 \geqslant 1$, so that the linear term in the density in Friedmann equation would be absent.

Obviously, models with analytical expansion, that is, natural exponents $\xi_0, \xi_1, \ldots$ (such as LCDM, for instance) do not show future singularities, neither weak nor strong.

These results are summarized in Table 3.

The normal branch of DGP cosmologies provide a relevant example for this section. If the bulk on which the brane lives is an anti-de Sitter spacetime one has

$$\frac{H}{H_0} = \sqrt{\frac{8G\pi\rho - |\Lambda_b|}{3H_0^2} + \Omega_{r_c}} - \sqrt{\Omega_{r_c}}. \qquad (20)$$

The identification with our perturbed formulation is given by $\rho_* = (|\Lambda_b| - 3H_0^2\Omega_{r_c})/(8\pi G)$, $h_0 = \Omega_{r_c}H_0^2$, $\xi_0 = 0$, $h_1 = -4H_0\sqrt{2\pi G\Omega_{r_c}}/(\sqrt{3})$ and $\xi_1 = 1/2$. This singularity is a sudden one, also referred to as quiescent [38], or using our terminology, it is a weak extremality event. A slight variation leading to a singularity of the same sort consists in letting the brane have a negative brane tension σ. In this case the bulk can either be the Minkowski or the

anti-de Sitter spacetime. The above expression can be adapted to this variation by simply letting $\rho \to \rho + \sigma$ and $\rho_* \to \rho_* + \sigma$.

Finally, we may consider models arising in loop quantum cosmology as those in [41], for which

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_*}\right),$$

but for these models the critical density is relevant for the high density regime, imposing a maximum density which is reached as the energy density grows. As this is the opposite of our working hypothesis (remember we demanded $\dot{\rho} < 0$) these models do not quite fit in our description here, but could be treated in an analogous way, with the corresponding adjustments.

3. Discussion

We here put forward a detailed classification of the future behavior of FRW cosmologies in modified gravity proposals. Departures from the standard description of the expansion of the Universe according to Einstein's theory have been considered of interest, as they could provide an explanation of what is the agent responsible for the accelerated expansion of the Universe.

The main question we pose is what are the characteristics of the modifications in connection with the presence of a singular future behavior of the Universe. As we have reflected here, not all the relevant properties of cosmic evolution emerge by considering curvature scalars, and the deeper insight provided by the consideration of observers is needed.

The spirit of the modified gravity proposals we consider is to assume the Universe is simply filled with cosmic dust, and no blueshifting component whatsoever is considered (unlike when one assumes the current cosmic acceleration is due to an exotic fluid or dark energy). Our starting point is a perturbative low-energy or infrared expansion of the modified Friedmann equation. Two classes emerge: those with a critical energy density and those without it. We find one has to consider at most the exponents of the first two terms of the expansion in order to differentiate the possible behaviors, and, more importantly, whether the future singularity, if it exists, is weak or strong.

The scheme we propose provides an easy route to conclude the sort of singular behavior present in potential new candidates to explain the current acceleration in the universe in terms of a modification of gravity. The classification we put forward is complementary to others, but provide a deeper insight and allow an important further degree of refinement.

Acknowledgements

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