Computer-Aided Design of Developable Surfaces: Designing with Developable Surfaces

Francisco Perez-Arribas*, Leonardo Fernandez-Jambrina
Naval Architecture School, Universidad Politécnica de Madrid (UPM), Spain.

* Corresponding author. Tel.: 913367140; email: francisco.perez.arribas@gmail.com
Manuscript submitted February 14, 2018; accepted May 30, 2018.
doi: 10.17706/jcp.13.10.1171-1176

Abstract: The use of developable surfaces in design addresses engineering needs because they can be easily manufactured without stretching or tearing, or without the use of heat treatment. In addition, they also require the minimum strain energy of flexure. This paper presents study cases of ship hulls that are of direct application on shipyards, although the method could be implemented in other fields, such as architecture or furniture design. Our approach is to create a developable B-spline surface between two boundary curves. Between each pair of these curves or directrix lines, the generatrix lines or rulings are created and a quasi-developable B-spline surface containing the rulings is defined. In engineering practice, quasi-developable surfaces can be considered as strictly developable ones, since a tolerance depending on the materials is used. The rulings are obtained by a searching technique based on the warp angle that depends on the material. The creation of a lofting B-spline surface containing the rulings enables the interchange between most common engineering softwares that work with NURBS (Non Uniform Rational B-Spline) surfaces by the use of IGES or STEP standard data exchange files. B-spline curves and surfaces are widely used today in practically all the design softwares. An example of ship hulls entirely created with developable surfaces is presented.

Key words: B-splines, developable surfaces, warp angle.

1. Introduction

Developable surfaces raise the interest of many different fields such as architecture, engineering, furniture or garments design. Their main property is that they can be unrolled or developed onto a plane without distortion and so, they can be constructed through the smooth bend of a plane sheet [1]. This kind of surfaces can be considered as a special case of ruled ones. Ruled surfaces are created by sweeping a straight line through the space along a directrix curve. To impose develop ability condition to this surface, the normal vector along each ruling should be constant. Thus, ruled surfaces can be classified as developable or warped ones regarding the position of the tangent plane to the surface, whose normal vector is constant along each ruling. In fact, for developable surfaces the tangent plane does not intersect the surface locally.

Many authors have studied the approximation of free form geometry by developable patches as [2]-[4]. Nevertheless, this approach is limited to certain kind of surfaces that already have a small Gaussian curvature value. Achieving developability by this method usually causes distortion in comparison with the original geometry.

This paper continues with a previous work by Perez *et al.* [5], based on creating a quasi-developable surface between a pair of boundary curves. This approach has been also studied by Aumman [1] defining a developable Bezier surface from two boundary curves and the rulings between pairs of points from each curve. Chalfant and Maekawa [6] extend Aumman's work including spatial curves that lie on non-parallel planes and Chu and Séquin [7] followed this approximation to create quadratic and cubic developable Bezier patches from two boundary surfaces. In the marine field, these methods have been applied to small craft design [8]. Recent works by Chen and Tang [9] have obtained a quasi-developable surface created from Bezier patches by the interpolation of two arbitrary space curves, including the properties of the materials to evaluate the developability degree.

The interest of this methodology is based on the workflow usually follow by ship designers. In fact, working with curves constitutes a simpler method to calculate and control many parameters that define the ship hull. Naval architects work directly with curves, as they associate different hydrodynamic properties to these geometries. The entrance and exit angles of water to the hull are also defined by curves. Moreover, these curves are finally translated into reinforcing structures in the construction of ship hulls.

On the other hand, as the materials for the construction of the ship hulls are usually plywood or metal rigid plates with a certain thickness, it is important to consider their physical properties. In the case of plywood hulls, for example, using non-developable surfaces could result on the internal damage of the material when forming and curving the surface, and so, its structural behaviour could be negatively affected.

In ship hull design, developable surfaces suppose great advantages as the hull construction usually involve materials in a sheet format such as plywood, steel or aluminium plates. Some boat hulls can be defined as developable surfaces and so, be built from flat sheets without stretching nor tearing them. Thus, working with this kind of surfaces requires the minimum strain energy of flexure and it can simplify an automated welding process [10] so it results in lower costs in construction and equipment and ease for repairing. In addition, because we part from sheet materials, waste is reduced as well as the costs for joining process [11].

2. Developable Surfaces

In a general case, a developable surface is the motion of a tangent plane of a curve in the space. The methodology we follow is based on [5] and it consists on this definition of developable surfaces applied to infinitesimal parts of the directrix curves. Thus, given a couple of curves s1 and s2 that constitute the directrix lines of the developable surface, the tangent planes T along the whole surface will be tangent to both lines. In addition, these planes will also contain a straight line di which is comprised in the surface (see Fig. 1). The ensembles of all the straight lines are the generators or the rulings of the developable surface. Hence, the normal vectors n1 and n2 to the tangent plane T at the endpoints of its corresponding ruling will be parallel.

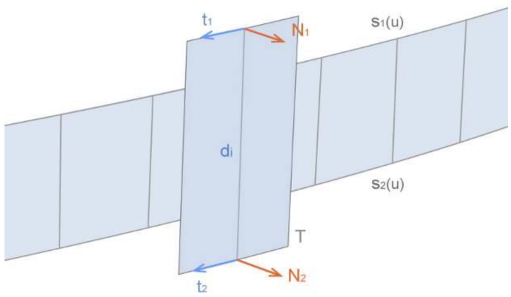

Fig. 1. Defining developability condition for a pair of boundary B-spline curves.

The normal vectors can be obtained from the following expressions:

$$\begin{aligned} n_1 &= r \times t_1 \\ n_2 &= r \times t_2 \end{aligned} \tag{1}$$

From a geometrical point of view, these two vectors will be parallel when their cross-product is equal to zero:

$$n_1 \times n_2 = 0 \tag{2}$$

And so, developability will be assured. However, in the presented methodology it is not necessary that this value is strictly equal to zero, but low enough. The module or length vector of the previous equation can be expressed as:

$$|n_1 \times n_2| = |n_1| \cdot |n_2| \cdot \sin(\Phi) \tag{3}$$

this results on the following expression when we work with unitary vectors:

$$|n_1 \times n_2| = \sin(\Phi) \tag{4}$$

This angle φ is called the warp angle and it can be defined as the angle that the tangent plane must warp to assure it is tangent to both directrix lines. The warp angle depends on material and other parameters and in practice, small values of this angle are allowed.

As we have mention previously, ship designers work indeed with curves, as it is the established method for defining small boat hull forms. These curves are the sheer line, the chines, and the center line of a ship hull and represent the directrix lines of a developable surface (See Fig. 2). Moreover, the character of these curves determines also the shape of the developable surface. So, if the directrices are designed smooth and continuous, the surface won't present any bumps and hollows, and it will be a faired surface.

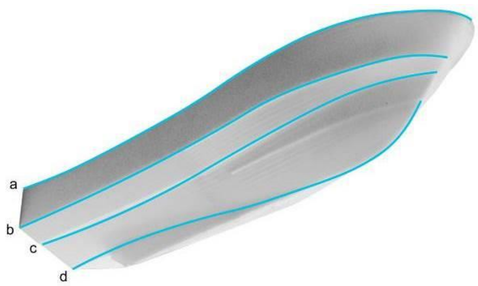

Fig. 2. B-spline curves represent the main curves of a ship hull: a) Sheer line b) Upper chine c) Lower chine d) Center line.

In CAD software, these directrices can be modeled as B-spline curves. We will usually work with cubic curves (n = 3) as this is the lowest degree that maintains the curvature between pieces of a B-spline. The designer should give a minimum number of control points to model the directrices. Thus, the first step is to model these B-spline curves and take advantage of its parametric definition. *A B*-spline curve of degree n has *N*+1 control points and it can be expressed as the following equation:

$$s(u) = \sum_{i=0}^{N} \mathbf{V_i} \cdot B_i^n(u) = \left(X(u), Y(u), Z(u) \right) = \sum_{i=0}^{N} \left(X_i \cdot B_i^n(u), Y_i \cdot B_i^n(u), Z_i \cdot B_i^n(u) \right) \tag{5}$$

where $Vi = (X_i, Y_i, Z_i)$ are the control points of the B-spline and B_i^n(u) are polynomial functions, both with (i = 0⋯N). $B_i^n(u)$ depends on a list of knots $\{u_{-1} \cdots u_{N+n}\}$ with $u_j \leq u_{j+1}$. The tangent vectors for any point of each directrix will be obtained by the derivatives s'(u) of the B-spline curve s(u):

$$s'(u)=\sum_{i=0}^{N}\mathbf{V}_i\cdot[B_i^n(u)]'=\sum_{i=0}^{N}\left(X_i\cdot[B_i^n(u)]',Y_i\cdot[B_i^n(u)]',Z_i\cdot[B_i^n(u)]'\right) \tag{6}$$

3. Finding the Rulings

The searching algorithm is based on the numerical approach defined in reference [5]. For each pair of boundary curves corresponding to the chines of the ship hull, we identify two directrices $s_1(u)$ and $s_2(u)$ of the developable surface that are parameterized as B-spline curves between the values 0 and 1. On the directrix $s_1(u)$, we choose a list of n fixed values of the parameter u_1. Each one corresponds to the start point of the ruling that we are searching and we can calculate its coordinates as well as its tangent vector to $s_1(u)$ with Eqs. (5) and (6). On the second directrix $s_2(u)$, the values of the parameter u2 are obtained by dividing the curve in m divisions with step size h. Each of these points is the end point of every potential ruling that starts on $s_1(u)$. Again, we use Eqs. (5) and (6) to obtain the coordinates of these points and the tangent vectors to $s_2(u)$ to each one. Once the tangent vectors t_1 and the multiple t_2 are obtained, we can generate the potential rulings r_i and so, calculate the normal vectors n_1 and n_2 with Eq. (1) (See Fig. 3).

The following step is the calculation of the cross-product of the normal vectors as we have established on Eq. (4). We obtain a list of values that represent the warp angle from where we select the minimum one. Finally, we retrieve the u_2 points on the curve $s_2(u)$ corresponding to the minimum warp angle. This search strategy is repeated for all the n values of the curve $s_1(u)$, obtaining a set of rulings that assure the minimum warp angle for each one (see Fig. 4).

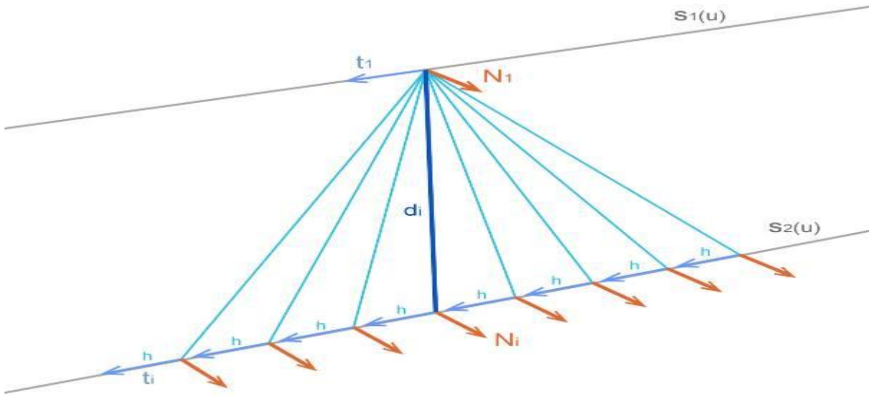

Fig. 3. Searching for the rulings method.

In all this procedure, it is important to remark that the original B-spline curves might be extended at their end points, as it is possible that there are no points u2 that solve the equation of developability for certain values u1 that lie near the end points of the first directrix. In this case, the search has to be extended to a bigger interval, which means the extension of the curve.

4. Example

One of the main properties of developable surfaces is that the Gaussian curvature is equal to zero. Developability can be numerically checked by conventional CAD tools analyzing the Gaussian curvature along a surface. If the created rulings present a certain tolerance for the cross-product of its normal vectors as described in previous sections, we can find a low Gaussian curvature value, but not exactly a zero one. Nevertheless, these quasi-developable surfaces can be considered developable regarding their feasibility to be constructed. In fact, almost any material has enough plasticity to admit slight deformation according to its warp angle.

One of the following steps of our research is to define warp angles and other properties according to different materials. From [10], we know that metals can stand a warp angle around six degrees, so if the warp angle of a metallic hull is below this limit, the surface can be considered developable. This example is described in [6] as a hard chine model. This hull is constructed from a couple of surfaces.

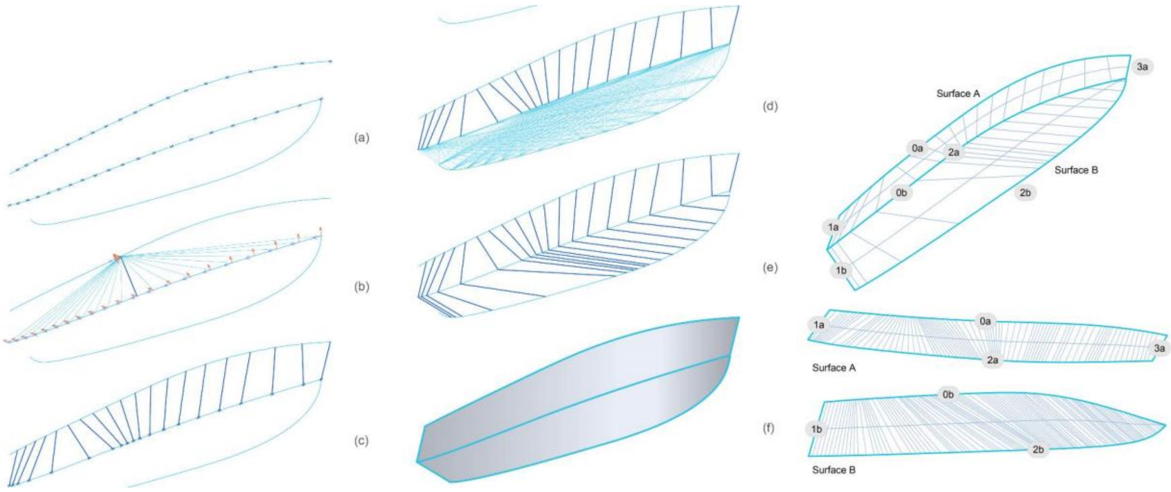

Fig. 4. a) Selection of points on curve s1(u) – the sheer line- and division of curve s2(u) – the chine- in n points. b) Application of the methodology for each point u1. c) The rulings in the first pair of boundary curves are obtained. d) The same method is followed for the center line and the chine. e) All the rulings are obtained for each pair of curves. f) The final surface is created after lofting the rulings.

5. Conclusions

This method is currently under development. From this first approach with visual programming (Grasshopper plug-in) we have already faced several limitations that will be solved in the next step while programming the final algorithm. However, this method offers a very intuitive and simplified way to obtain the rulings and the developable surface that contains two boundary curves. Another advantage of this method is its versatility to import and export data and geometry to other specific programs and formats for engineering such as IGES or STEP.

From a practical point of view, it is important to focus on the warp angle to obtain the tolerance for developability depending on the physical properties of the material. This will lead us to a physic/geometric algorithm instead of numerical one, which is the main difference with the approaches in this field made by other authors.

At this point, we propose a further step of the original methodology to develop the algorithm taking into account the warp angle as a variable: to consider warp angle below the limit value of the material instead of a minimal one. With this technique some parts of the area of regression will be eliminated. A final step will be the modification of the control points of one of the directrix lines in order to adjust its shape, with the use of optimization algorithms that minimize the geometric warp angle.

Acknowledgment

This work is partially supported by the Spanish Ministerio de Economía y Competitividad through research grant TRA2015-67788-P: "Generación Paramétrica de Formas de Embarcaciones Rápidas mediante superficies NURBS para su Optimización Hidrodinámica".

References

[1] Azariadis, P. N., & Sapidis, N. S. (2003). Planar development of digital free-form surfaces. In M. Sarfraz

(Ed.). *Advances in Geometric Modeling*. Chichester, UK: John Wiley & Sons, Ltd.

[2] Julius, D., Kraevoy, V., & Sheffer, A. (2005). D-charts: Quasi-developable mesh segmentation. *Eurographics*, *24(3),* 581-590.

[3] Wang, C. (2007). Computing length-preserved free boundary for quasi-developable mesh segmentation. *IEEE Transactions on Visualization & Computer Graphics*, *14,* 25-36.

[4] Fernández-Jambrina, L. (2017). Bézier developable surfaces. *Computer Aided Geometric Design*, *55,* 15-28.

[5] Perez, F., & Suarez, J. A. (2007). Quasi-developable B-spline surfaces in ship hull design. *Computer-Aided Design*, *39(10),* 853-862.

[6] Chalfant, J. S., & Maekawa, T. (1998). Design for manufacturing using b-spline developable surfaces. *Journal of Ship Research*, *42(3),* 207-215.

[7] Chu, C. H., & Séquin, C. H. (2002). Developable Bézier patches: Properties and design. *Computer-Aided Design*, *34(7),* 511-527.

[8] Nolan, T. J. (1971). Computer-aided design of developable hull surfaces. *Maritime Technology*, 233-242.

[9] Fan, S. K., & Joneja, A. (2015). Geometric optimization for developable panel approximation. *Computer-Aided Design and Applications*, *12(2),* 147-155.

[10] Barry, C. (2001). Working with developable surfaces. *Boatbuilder*, 800-786-3459.

[11] Calkins, D. E., Theodoracatos, V., & Guilar, G. D. (1989). Small craft hull form surface definition in a high-level computer graphics design environment. *SNAME Transactions*, *97,* 85-113.

Francisco Pérez-Arribas is an associate professor in the Naval Architecture and Marine Engineering School of Madrid (ETSIN), UPM. He received BS and MS degrees in naval architecture from the Universidad Politécnica de Madrid. He has working experience at the ETSIN Towing Tank in CAD and CASHD. His current research interests include ship hull modelling, parametric ship design and geometric modelling.

Leonardo Fernández-Jambrina is a professor of applied maths at the Universidad Politécnica de Madrid, Spain, since 2000. He has worked previously as researcher and assistant professor at the Universidad Complutense de Madrid, where he received his bachelor of science degrees both on mathematics and physics and his Ph.D on physics. His current research interests include computer-aided design and geometric modeling with applications to naval architecture.