

Hidden past of dark energy cosmological models

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Abstract

In this Letter we analyse the possibility of having homogeneous isotropic cosmological models with observers reaching $t = \infty$ in finite proper time. It is shown that just observationally-suggested dark energy models with $w \in (-5/3, -1)$ show this feature and that they are endowed with an exotic curvature singularity. Furthermore, it is shown that non-accelerated observers in these models may experience a duration of the universe as short as desired by increasing their linear momentum. A subdivision of phantom models in two families according to this behavior is suggested. © 2007 Elsevier B.V. All rights reserved.

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1. Introduction

During the last years there has been mounting experimental evidence from different sources (supernovae type Ia [1], redshift of distant objects [2] and temperature fluctuations of background radiation [3]) supporting an accelerated expansion of our Universe at present time (cf. for instance [4] for a review).

Trying to explain this fact, several proposals have been made, such as dark energy contents for the universe or modifications of the theory of gravity, which have produced a menagerie of new types of singular events in the respective cosmological models, traditionally restricted to Big Bang and Big Crunch singularities. For instance, we may find in phantom energy models Big Rip singularities [5]. An attempt to explain the accelerated expansion without violating all energy conditions [6] produces sudden singularities. Most recently, inaccessible singularities in toral cosmologies have been added to the list [7].

There have been several attempts to organize these families of singular events in thorough classifications. In [8] all types of singular events in Friedmann–Lemaître–Robertson–Walker (FLRW) cosmological models are classified according to the

coefficients and exponents of a power expansion in time of the scale factor of the universe around the event. In [9] singularities are classified using the finiteness of the scale factor, the density and the pressure of the universe. In [10] the behavior of causal geodesics close to singular events and the strength of the singularities are analysed.

This line of research has proven successful showing unexpected features of FLRW cosmological models near the singularities. For instance, it has allowed to show that sudden singularities are weak [11], since tidal forces do not disrupt finite objects falling into them [12–14].

Another intriguing feature concerning Big Rip singularities is that photons do not experience such fate for effective equations of state, $p = w\rho$, with $w \in (-5/3, -1)$ (that is, those comprised between the superphantom [15] and the phantom divide), since they require an infinite lapse of time to reach that event [10]. Since this range of the parameter w comprises the observationally accepted values [16], which are slightly below -1 , this fact is more than a mere curiosity.

Following the idea of classifying the singular events arising in FLRW cosmological models, it is worth mentioning that all classifications are incomplete in a sense: they unveil what happens at a finite coordinate time t , but they are elusive when asked about infinite t . This may seem a pointless consideration, since in most cases an infinite coordinate time lapse corresponds to an infinite time lapse experienced by the observer, but

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the mentioned example about photons in phantom cosmologies, where a finite coordinate time lapse requires an infinite proper time shows us that the issue is far from being trivial.

To this aim in Section 2 the equations governing causal geodesics in FLRW cosmological models are reviewed. In Section 3 the conditions for a causal geodesic to reach $t = \pm\infty$ in finite proper time are derived. It will be shown that just phantom models fulfill this property. In Section 4 it will be discussed if this abrupt end of causal geodesics is an actual singularity or not. Analysis of the Ricci curvature as measured by the observers will settle the issue, in spite of the zero value of curvature scalar polynomials there. In fact these are strong curvature singularities. Finally, the consequences of these facts will be discussed in Section 5.

2. Geodesics in FLRW cosmological models

The metric for FLRW cosmological models may be written,

$$ds^2 = -dt^2 + a^2(t) \{ f^2(r) dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \},$$

$$f(r) = \frac{1}{\sqrt{1 - kr^2}}, \quad k = 0, \pm 1, \quad (1)$$

in terms of spherical coordinates r, θ, ϕ with their usual ranges and a coordinate time, with a range depending on the type of cosmological model.

Three families of models are comprised in this expression, open models with $k = -1$, flat models with $k = 0$ and closed models with $k = 1$. Observations favor flat models, but we keep for our purposes the general formula.

Free-falling observers in a spacetime are modeled by time-like geodesics parametrized by proper time τ , since these curves have the property of vanishing acceleration. The use of proper time allows us to write the velocity u of the parametrization of the geodesic $(\dot{t}, \dot{r}, \dot{\theta}, \dot{\phi})$ as a unitary vector,

$$\delta = -g_{ij} \dot{x}^i \dot{x}^j, \quad x^i, x^j = t, r, \theta, \phi, \quad (2)$$

where the dot means derivation with respect to τ .

There are three types of geodesics: timelike ($\delta = 1$), spacelike ($\delta = -1$) and lightlike ($\delta = 0$). We consider just causal geodesics, $\delta = 0, 1$, since they are the only ones that may carry signals or observers.

A quick way to write down simple geodesic equations for these spacetimes is taking into account that the universe is homogeneous and isotropic and therefore geodesics are straight lines in the spacetime and we may take $\dot{\theta} = 0 = \dot{\phi}$ without loss of generality.

It is also easy to check that the vector $\partial_R = \partial_r / f(r)$,

$$R = \begin{cases} \operatorname{arcsinh} r, & k = -1, \\ r, & k = 0, \\ \operatorname{arcsin} r, & k = 1, \end{cases}$$

generates an isometry along these straight lines and therefore there is a conserved quantity P of geodesic motion attached to it, the specific linear momentum of the observer,

$$\pm P = u \cdot \partial_R = a^2(t) f(r) \dot{r}, \quad (3)$$

where the \cdot denotes the inner product defined by the metric (1). The double sign is introduced in order to keep P positive.

We just need another equation for \dot{t} to complete the set and we may obtain it without resorting to Christoffel symbols for the metric by using the unitarity condition (2),

$$\delta = \dot{t}^2 - a^2(t) f^2(r) \dot{r}^2.$$

Restricting to future-pointing geodesics, $\dot{t} > 0$ (past-pointing geodesics are treated in a similar fashion), the whole set of geodesic equations is reduced to

$$\dot{t} = \sqrt{\delta + \frac{P^2}{a^2(t)}}, \quad (4a)$$

$$\dot{r} = \pm \frac{P}{a^2(t) f(r)}. \quad (4b)$$

Hence we see that there are basically three types of causal geodesics: radial lightlike ($\delta = 0, P \neq 0$) and timelike geodesics ($\delta = 1, P \neq 0$) and the comoving congruence of fluid worldlines ($\delta = 1, P = 0$), which provide little information about the geometry of spacetime, since for them $t = \tau$ regardless of the possible singularities in the universe.

3. Singularities at infinity

Since singularities along causal geodesics at a finite t_0 were considered in detail in [10], we focus now on infinite values of coordinate time t .

Singularities may appear also at $t = \pm\infty$ if there are observers that reach these events in finite proper time. Unfortunately, it is not always possible to perform power expansions of the scale factor centered in $t = \pm\infty$, as it is done in [8,10] for finite t , since there are physically reasonable spacetimes with oscillatory scale factors, for instance, anti-de Sitter universes, for which the limit of $a(t)$ is not defined when t tends to infinity.

However, the question of when $t = \pm\infty$ is reached by geodesic observers in finite proper time can be easily solved.

For lightlike radial geodesics we have

$$\dot{t} = \frac{P}{a(t)},$$

$$\int_{t_0}^t a(t') dt' = P(\tau - \tau_0),$$

and therefore lightlike geodesics reach $t = \infty$ in finite proper time if and only if the integral

$$\int_t^\infty a(t') dt' \quad (5)$$

is finite for sufficiently large t . That is, if $a(t)$ is an integrable function at infinity.

Comoving fluid worldlines with $P = 0$ need not be considered, since they reach $t = \infty$ in infinite proper time.

Finally, we have timelike radial geodesics. In this case, proper time may be written again in terms of an integral of $a(t)$

using (4a),

$$\int_{t_0}^t \frac{dt'}{\sqrt{1 + P^2/a^2(t')}} = \tau - \tau_0, \quad (6)$$

and therefore these geodesics reach $t = \infty$ in finite proper time if and only if the improper integral

$$\int_t^\infty \frac{dt'}{\sqrt{1 + P^2/a^2(t')}}$$

is convergent for sufficiently large t .

Obviously this can only happen if $a(t)$ tends to zero at infinity, but it is not a sufficient condition. Since we may bound

$$\int_t^\infty \frac{dt'}{\sqrt{1 + P^2/a^2(t')}} < \frac{1}{P} \int_t^\infty a(t') dt',$$

the integral for timelike geodesics is convergent if the one for lightlike geodesics is.

Furthermore, since for large t and $a(t)$ tending to zero

$$\frac{1}{\sqrt{1 + P^2/a^2(t')}} = \frac{a(t')}{P} - \frac{1}{2} \frac{a^3(t')}{P^3} + \dots,$$

is a telescopic series, the integral for timelike geodesics converges if and only if the one for lightlike geodesics does. Hence, all radial geodesics have the same regularity pattern.

The analysis for $t = -\infty$ is entirely similar and so we have focused on the $t = \infty$ case.

Since in most models the scale factor $a(t)$ behaves asymptotically as a power of coordinate time, we start considering scale factors which behave close to infinity as

$$a(t) \simeq c|t|^\eta, \quad c > 0, \quad w = \frac{2}{3\eta} - 1.$$

The equation for lightlike geodesics (4a) may be integrated close to infinity,

$$t \simeq \left\{ \frac{(1 + \eta)P}{c} \right\}^{1/(1+\eta)} (\tau - \tau_0)^{1/(1+\eta)}, \quad \eta \neq -1,$$

$$t \simeq e^{P(\tau - \tau_0)/c}, \quad \eta = -1,$$

and provides valuable information, since t diverges when τ tends to infinity for $\eta \geq -1$, whereas t diverges at finite proper time τ_0 if $\eta < -1$.

The latter cases are quite interesting, since at τ_0 the geodesic reaches $t = \infty$ in finite proper time. Therefore, lightlike geodesics range from $\tau = -\infty$ ($t = 0$) to $\tau = \tau_0$ ($t = \infty$) and are incomplete towards the future.

This is not the interesting case, since it involves models starting at a Big Rip at $t = 0$. But if we consider $t = -\infty$, lightlike geodesics range from $\tau = \tau_0$ ($t = -\infty$) to $\tau = \infty$ ($t = 0$) and are incomplete towards the past. This is the usual range in the suggested phantom models.

As it has been said, the same behavior appears for timelike radial geodesics, with the difference that these actually end up at the Big Rip at $t = 0$ in a finite proper time [10].

Not only causal geodesics, but also spatial geodesics show this feature.

For non-tilted spatial geodesics in a hypersurface $t = t_0$,

$$\dot{i} = 0 \Rightarrow t = t_0, \quad P = a(t_0),$$

$$\dot{r} = \pm \frac{1}{a(t_0)f(r)} \Rightarrow R = \pm \frac{s - s_0}{a(t_0)},$$

proper distance s is essentially the radial coordinate R , corrected by the expansion factor, as expected.

But for tilted spatial geodesics,

$$\dot{i} = \sqrt{\frac{P^2}{a^2(t)} - 1} \Rightarrow s - s_0 = \int_t^\infty \frac{dt'}{\sqrt{P^2/a^2(t') - 1}},$$

and for $a(t) \simeq c|t|^\eta$ for large t , this integral converges to a finite value if and only if $\eta < -1$. Hence the length of these tilted spatial geodesics is also finite, even though the radial coordinate r diverges.

4. Curvature singularities

However, at $t = \pm\infty$ all curvature scalar polynomials vanish, since they decrease as t^{-2} and this suggests a sort of Minkowskian limit. Therefore there is no scalar polynomial curvature singularity there. A pathological feature named imprisoned incompleteness, which appears in spacetimes like Taub-NUT, where geodesics are incomplete without singular scalars of curvature, is not feasible, since the spacetime has a cosmic time (a function with timelike gradient everywhere, the coordinate time t) and is therefore causally stable [17,18]. We might suspect that geodesic incompleteness could simply point out that the spacetime is not fully covered with the coordinate patch (1) and that therefore it could be extendible beyond $t = \pm\infty$.

This is the case, for instance, of Milne universe, corresponding to $k = -1$, $a(t) = t$ in (1). A suitable coordinate transformation

$$T = t\sqrt{1 + r^2}, \quad R = rt,$$

shows that this model is just the portion of Minkowski spacetime inside the null cone $T = R$ and therefore the apparent singularity at $t = 0$ is due just to the choice of coordinates. Or for Schwarzschild spacetime, which in Schwarzschild coordinates appears to be singular at the horizon at $r = 2M$, whereas this coordinate singularity disappears on extending it with Eddington–Finkelstein [19] or Kruskal [20] coordinates.

However, the null value of the scale factor in that limit suggests a point as a limit in this case.

If we compute the Ricci tensor component along the velocity of the geodesic, an exotic behavior appears.

For a radial lightlike geodesic,

$$u^t = \dot{t} = \frac{P}{a}, \quad u^r = \dot{r} = \pm \frac{P}{fa^2},$$

$$R_{ij}u^i u^j = 2P^2 \left(\frac{a'^2 + k}{a^4} - \frac{a''}{a^3} \right) \simeq \frac{2P^2\eta}{c^2 t^{2(\eta+1)}} + \frac{2kP^2}{c^4 t^{4\eta}}, \quad (7)$$

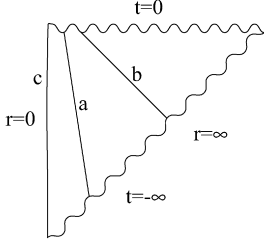


Fig. 1. Conformal diagram for a model with $w \in (-5/3, -1)$: Timelike radial geodesics like a have finite length, whereas lightlike geodesics like b are infinite towards the future and timelike geodesics like c are infinite towards the past.

we take a look at the first term of the zero component of the Ricci curvature, since it is present regardless of the value of k ,

$$2P^2 \left(\frac{a'^2}{a^4} - \frac{a''}{a^3} \right) \simeq \frac{2P^2 \eta}{c^2 t^{2(\eta+1)}} \simeq \frac{2\eta}{(\eta+1)^2} \frac{1}{(\tau - \tau_0)^2}, \quad (8)$$

and find out that the Ricci curvature diverges when t approaches $\pm\infty$ (τ tends to τ_0) for $\eta < -1$.

A similar analysis may be performed for radial timelike geodesics,

$$u^t = \dot{t} = \sqrt{1 + \frac{P^2}{a^2}}, \quad u^r = \dot{r} = \pm \frac{P}{fa^2},$$

$$R_{ij} u^i u^j = -\frac{3a''}{a} + 2P^2 \left(\frac{a'^2 + k}{a^4} - \frac{a''}{a^3} \right), \quad (9)$$

since for scale factors with $\eta < -1$ and large values of $|t|$ the dominant term is the P^2 -term, which is the same as for lightlike geodesics.

Hence the Ricci curvature diverges along both families of radial geodesics on approaching $t = \pm\infty$.

Hence we are to conclude that universes with scale factor $a(t) \simeq c|t|^\eta$, $\eta < -1$, for large values of $|t|$ have a p.p. curvature singularity (curvature singularity along a parallelly-transported basis) [17] at $t = \pm\infty$, though the scalar polynomials of curvature are zero there.

That is, there is an *actual* curvature singularity at $t = \pm\infty$ for these models, which correspond to $w \in (-5/3, -1)$, which is reached by the observers in finite proper time. Considering just expanding models of this type, all radial observers would trace their geodesic paths from the initial singularity at $t = -\infty$ to the Big Rip at $t = 0$ in a finite lapse of proper time.

This result does not contradict the Penrose diagrams for these models shown in [21], since conformal diagrams provide no information about distances, just about angles, but introduces a difference between models with $w \in (-5/3, -1)$ and those with $w \leq -5/3$ as it is shown in Fig. 1.

Furthermore, we may check the strength of these curvature singularities, which might be relevant, since other types of singularities, such as sudden singularities [6] (singularities II and IV in [9]) were shown not to be strong enough to disrupt finite objects [11] and even have been suggested to be consistent with observations [22].

Definitions of singularities related to curvature and geodesics refer to ideal point observers. When finite objects are con-

sidered, tidal forces are relevant and it is interesting to check if they may destroy the object. In this case the singularity is considered to be strong [12]. This qualitative concept has been stated rigorously by several authors [13,14,23,24].

For instance, in Tipler's definition [13] a curvature singularity is strong if the volume spanned by three Jacobi fields referred to an orthonormal basis parallelly-transported along a causal geodesic tends to zero at the singularity. Królak's definition [14] just requires that the derivative of this volume be negative.

There are necessary and sufficient conditions for the appearance of strong singularities [25], that become quite simple to implement in the case of FLRW spacetimes, since the Weyl tensor vanishes [10].

With Tipler's definition a lightlike geodesic of velocity u comes up a strong singularity at τ_0 if and only if the integral

$$\int_0^\tau d\tau' \int_0^{\tau'} d\tau'' R_{ij} u^i u^j \quad (10)$$

diverges as τ tends to τ_0 .

And with Królak's definition a lightlike geodesic of velocity u comes up a strong singularity at τ_0 if and only if

$$\int_0^\tau d\tau' R_{ij} u^i u^j \quad (11)$$

diverges as τ tends to τ_0 .

Since the Ricci curvature component (7) diverges as $1/(\tau - \tau_0)^2$, the integral of this term provides a logarithmic divergence at τ_0 for $\eta < -1$ with Tipler's definition and an inverse power divergence with Królak's definition. Therefore, lightlike geodesics meet a strong singularity at $t = \pm\infty$ if and only if $\eta < -1$. The contribution of the curvature term diverges even faster when present.

The previous conditions on integrals of Ricci components become sufficient conditions on dealing with timelike geodesics. Since the behavior of Ricci curvature has been shown to be similar for both families of radial geodesics for large $|t|$ and $\eta < -1$, we learn that also radial timelike geodesics meet a strong curvature singularity at $t = \pm\infty$.

We may easily extend this result to non-power law growth/decrease of the expansion factor:

- For $a(t)$ growing or decreasing as $1/|t|$ or slower, radial geodesics reach $t = \pm\infty$ in infinite proper time.
- For $a(t)$ decreasing faster than $1/|t|$, radial geodesics reach $t = \pm\infty$ in finite proper time and therefore there is an actual strong curvature singularity there, except for the de Sitter spacetime.

These two cases include all situations for which the scale factor has a well-defined limit $t \rightarrow \pm\infty$. Oscillatory scale factors may be treated directly with condition (5).

5. Discussion

So far we have shown that FLRW cosmological models for which $a(t)$ decreases faster than $1/|t|$ for large values of $|t|$ show a strong curvature singularity for $t \rightarrow \pm\infty$. This is the case of phantom models with $w \in (-5/3, -1)$, family that includes models compatible with observations, since w is estimated to be slightly below minus one [16].

Since the implications of these results are related to the past of the models instead of their future, it might seem a pointless discussion, for phantom models are intended to describe the future of the universe from now on. In the past other fields such as dust, radiation and the cosmological constant would be dominant and would prevent the appearance of the exotic curvature singularities described here.

However, even though phantom models are not relevant to study the past of the universe, there are still consequences that are applicable to our present Universe.

We may consider, for instance, the total duration of a universe filled with a phantom field as experienced by a free-falling observer (6),

$$\begin{aligned} T &= \int_{-\infty}^0 \frac{dt}{\sqrt{1 + P^2/a^2(t)}} = \int_{-\infty}^0 \frac{dt}{\sqrt{1 + P^2/c^2 t^{2\eta}}} \\ &= \left(\frac{P}{c}\right)^{1/\eta} \int_0^{\infty} \frac{x^\eta dx}{\sqrt{1 + x^{2\eta}}} \\ &= -\left(\frac{P}{c}\right)^{1/\eta} \frac{B(-\frac{1}{2\eta}, \frac{1}{2} + \frac{1}{2\eta})}{2\eta}, \quad \eta < -1, \end{aligned} \quad (12)$$

by the change of variable $x = -(c/P)^{1/\eta} t$, using the hypergeometric function Beta.

We already know that this expression for the time span is finite for $\eta < -1$, but it can be made as small as desired by taking arbitrary large values of the linear momentum of the observer P . There is no lower bound, nor upper bound, which we know it is infinite for non-radial observers.

Though the calculation has been made for the duration of the universe from the initial singularity to the Big Rip, it is clear that this result is also valid for the time span from the coincidence moment when phantom fields become the dominant component of the content of the universe to the Big Rip. That is, non-accelerated observers may shorten the time span to the end of the universe at will by increasing their linear momentum P .

This feature is exclusive of dark energy models with $w \in (-5/3, -1)$, since a negative exponent η is required in (12) for the decreasing behavior of T . For models with $w \leq -5/3$, $\eta \in [-1, 0)$ the integral (12) is divergent, since $t = -\infty$ is actually at infinity, but the decreasing behavior is also exhibited for finite intervals of time up to the Big Rip singularity, though the interest on these models is so far quite limited.

Another issue is the character of the singularity. Since $a(t)$ tends to zero at $t = \pm\infty$ for models with $w \in (-5/3, -1)$, this might suggest a sort of Big Bang singularity, though endowed with exotic features. However, the sign of the Ricci curvature

measured by causal geodesics (7), (9) prevents this interpretation, since in (8) we see that it is negative (non-focusing) for negative η in flat models.

In fact, for radial geodesics,

$$\begin{aligned} \frac{dR}{dt} &= \pm \frac{\dot{R}}{t} = \pm \frac{P}{a(t)\sqrt{P^2 + a^2(t)}} \simeq \pm \frac{1}{a(t)}, \\ R &\simeq R_0 \pm \frac{1}{c} \frac{t^{1-\eta}}{1-\eta}, \end{aligned}$$

the radial coordinate diverges for large $|t|$ in models with $\eta < 1$, and shows that geodesics are indeed not focusing and therefore geodesics diverge instead of converge.

With all these results in mind, we may refine the usual classification of flat models according to the value of w by introducing this new information, bearing in mind that none of these models is valid for the whole life of the universe, just for a fraction of it:

1. Models with $\eta > 1$, $-1 < w < -1/3$: Quintessence models with a Big Bang singularity at $t = 0$.
2. Models with $\eta = 1$, $k = -1$, $w = -1/3$: Milne-like models which may have weak or strong singularities at $t = 0$ [8,10].
3. Models with $0 < \eta < 1$, $w > -1/3$: Classical models (dust, radiation, ...) with a Big Bang singularity at $t = 0$.
4. Models with $\eta = 0$: The menagerie of models which are either regular (de Sitter, anti-de Sitter, ...) or possess sudden, freeze, pressure, higher derivative singularities as described in [8–10], which may be weak or strong.
5. Models with $-1 \leq \eta < 0$, $w \leq -5/3$: Phantom models ranging from $t = -\infty$ to $t = 0$ where they meet a Big Rip singularity. Scalar perturbations of these models have been shown to lead to high inhomogeneity, which may prevent the formation of the singularity [26].
6. Models with $\eta < -1$, $w \in (-5/3, -1)$: Phantom models with a p.p. curvature singularity at $t = -\infty$ which affects just radial geodesics and a Big Rip singularity at $t = 0$ which does not show up for lightlike geodesics. One of these phantom models has been shown to be stable against quantum corrections [27].

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