

Number Theory. Program 2000/2001.

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Chapter 1. **Divisibility**

- 1.1 Polygonal numbers.
- 1.2 Divisibility.
- 1.3 The existence of infinitely many prime numbers.
- 1.4 Arithmetic functions.
- 1.5 Generating functions.

Chapter 2. **Congruences**

- 2.1 Definition and properties.
- 2.2 Linear congruences.
- 2.3 Simultaneous congruences.
- 2.4 Polynomial congruences. Lagrange's Theorem.
- 2.5 Quadratic residues.
- 2.6 Legendre and Jacobi symbols.
- 2.7 Quadratic reciprocity law.
- 2.8 Primitive roots.

Chapter 3. **The distribution of prime numbers.**

- 3.1 Erathostenes' sieve.
- 3.2 The functions $\pi(x)$, $\psi(x)$ and $\theta(x)$.
- 3.3 Tchebychev's Theorem.
- 3.4 Bertrand's Postulate.
- 3.5 Mertens' Formulae.

3.6 The prime number Theorem.

3.7 Relation between $\psi(x)$ and the zeroes of $\zeta(s)$.

Chapter 4. **Primes in arithmetic progressions.**

4.1 Definition and examples of characters.

4.2 Construction of characters for general modulus.

4.3 Orthogonality

4.4 Primitive characters.

4.5 Sums of characters.

4.6 Gauss' sums for prime modulus.

4.7 Quadratic residues and cyclotomy.

4.8 Dirichlet's Theorem.

Chapter 5. **Diophantine analysis.**

5.1 Farey fractions.

5.2 Dirichlet's Theorem for irrationality.

5.3 Sequences uniformly distributed, Weyl's Theorem.

5.4 Continued fractions.

5.5 Hurwitz's Theorem.

5.6 Pythagorean triples and Pell's equation.

5.7 k -powers. Lagrange and Fermat.

References

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