Quasiconformal mappings, elliptic equations and beyond (Long Agapi Days)

Instituto de Ciencias Matemáticas (Madrid)

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Title

From Teichmüller to Shoen–Yau : Extremal mappings between Riemann surfaces.

Gaven Martin, Massey University.

Abstract

There are two now classical descriptions of the moduli space of a Riemann surface via the theory of extremal mappings. The first from Teichmüller in the 1940s (rigorously established by Ahlfors in 1953) and through the existence of extremal quasiconformal mappings. The second is through Schoen-Yau's existence theory for unique harmonic diffeomorphisms in the 1970s, and developed into a theory of moduli by many, including Wolf, Tromba and Wolpert many years later. The important ingredient in both is the existence of a holomorphic quadratic differential, from the Beltrami coefficient of an extremal quasiconformal mapping (Teichmüller) or from the Hopf equation (Harmonic). These quadratic differentials define the cotangent space to the moduli space. Here we show that in fact both of these approaches are manifestations of the same theory (that of existence of diffeomorphic extremal mappings of finite distortion) in limiting regimes. We identify parameterised families of moduli spaces (Beltrami coefficients) interpolating between these two end cases defined by a parametrised family of degenerate elliptic nonlinear PDEs giving holomorphically parameterised homotopy between the extremal quasiconformal mapping [which is not a diffeomorphism] and the harmonic diffeomorphism.

Title
Drilling groups and Cannon's conjecture.

Peter Haïssinsky, Université d'Aix-Marseille.
Joint work with **D. Groves, J. Manning, D. Osajda, A. Sisto and G. Walsh.**

Abstract

Drilling consists traditionally in removing a simple closed geodesic of a hyperbolic 3-manifold and endowing its complement with a new complete hyperbolic structure. In this talk, I will explain how drilling can be adapted to the context of groups and how it provides a new approach to the dynamical characterisation of cocompact Kleinian groups (Cannon's conjecture).

Title
Geometry of domains and harmonic quasiconformal mappings.

Vesna Todorcevic, Faculty of Organizational Sciences and Mathematical Institute SASA.

Abstract

We will consider several topics related to harmonic quasiconformal maps. In particular we will discuss generalizations of some classical results related to geometry of domains, arising from the work of Astala, Zinsmeister and others as well as some joint work of the author and Kari Astala.

Title
Convex integration and compensated compactness quantities in ideal magnetohydrodynamics.

Sauli Lindberg, University of Helsinki.

Abstract

The talk gives an overview of my collaboration with Daniel Faraco and László Székelyhidi Jr. on classical conservation laws of ideal magnetohydrodynamics (MHD). I will highlight the decisive role of certain compensated compactness quantities in ideal MHD. I will also sketch some ideas of our convex integration constructions in ideal MHD. They are used to prove (in accordance with simulations) that there exist bounded, energy dissipative weak solutions.

Title
On the Principle of Non-interpenetration of Matter.

Tadeuz Iwaniec, Syracuse University.

Abstract

It is a common struggle in mathematical models of Nonlinear Hyperelasticity to establish existence of the energy-minimal deformations that are invertible. This is a fundamental prerequisite when modeling the deformation response of solid materials. This requirement for hyper-elastic bodies was first studied by J. Ball (1981) by means of Global Inversion Theorem. Direct method in the Calculus of Variations reveals that injectivity can be lost when passing to the limit of an energy-minimizing sequence of homeomorphisms. Simply, some parts of the material body (with positive volume) can be squeezed to lower dimensional pieces in the deformed configuration. From the topological point of view this squeezing incident is characteristic of the so-called Monotone Deformations, which turn out to be $W^{1,p}$-limits of homeomorphisms. It is for this reason that we shall accept Sobolev Monotone Mappings as legitimate deformations when modeling (mathematically reliable) Theory of Hyperelasticity.

Title
On a class of random analytic functions.

Eero Saksman, Helsinki University.

Abstract

We consider a class of random inner functions related to GMC (Gaussian multiplicative chaos) measures. The talk is based on a joint work with **Yichao Huang** (Peking).

Title
Dirichlet space over Jordan domains.

Michel Zinsmeister, Université d'Orléans.
Joint work with **Wei Huaying**, Jiangsu Normal University at Xuzhou.

Abstract

If U be a C^1 function with compact support in the plane and u its restriction to the unit circle $\mathbb{S}$, let us denote by U_i, U_e the harmonic extensions of u respectively in the interior and the exterior of $\mathbb{S}$ in the Riemann sphere. About a hundred years ago, Jesse Douglas, for his solution of the Plateau problem, has shown that

$$\begin{aligned}\iint_{\mathbb{D}} |\nabla U_i|^2(z)dxdy &= \iint_{\bar{\mathbb{C}}\setminus\bar{\mathbb{D}}} |\nabla U_e|^2(z)dxdy \\ &= \frac{1}{2\pi}\iint_{\mathbb{S}\times\mathbb{S}} \left|\frac{u(z_1)-u(z_2)}{z_1-z_2}\right|^2 |dz_1||dz_2|,\end{aligned}$$

thus giving three ways to express what is called the Dirichlet norm of u. The main goal of this talk is to give a characterization of rectifiable Jordan curves for which the obvious analogue of these three (semi-)norms are equivalent.

Title

Energy of a quasiconformal mapping and the intrinsic distance.

Pekka Koskela, (University of Jyväskylä).

Abstract

The membership of a quasiconformal mapping, defined in the unit ball, in a Hardy space is equivalent to the mapping having finite energy in a suitable sense. We give an intrinsic version of this statement in a more general case.

Title
Hardy spaces and quasiregular mappings.

Maria José González Fuentes, Universidad de Cadiz.

Abstract

We study Hardy spaces for quasiregular mappings on the unit ball which satisfy appropriate growth and multiplicity conditions. Under these conditions we recover several classical results for analytic functions. This program was initiated and developed for Hardy spaces of quasiconformal mappings by K. Astala and P. Koskela in 2011 in their paper Hp-theory for Quasiconformal Mappings. (Joint work with T. Adamowicz).

Title
Sobolev flows of non-Lipschitz vector fields.

Albert Clop, Universitat de Barcelona.

Abstract

In contrast to the classical Picard Theorem, for which the flow of a Lipschitz vector field is spatially bilipschitz, recent works by Jabin, or also by Alberti – Crippa – Mazzucato, show that flows arising from DiPerna – Lions theory may exhibit dramatic losses of regularity and enjoy no Sobolev smoothness, even of fractional order. Between these two extreme situations, one can find many vector fields arising from Geometric Function Theory or also from Fluid Mechanics. In this talk we will review these examples and explore what can be done to understand the Sobolev smoothness of their flows.

Title

Homeomorphic Sobolev extensions of parametrizations of Jordan curves.

Jarmo Jääskeläinen, Helsinki University.
Joint work with **Bouchala, Koskela, Xu and Zhou.**

Abstract

Each homeomorphic paramatrization of a Jordan curve via the unit circle extends to a homeomorphism of the entire plane. It is a natural question to ask if such a homeomorphism can have some Sobolev regularity. This prompts the simplified question : for a homeomorphic embedding of the unit circle into the plane, when can we find a homeomorphism from the unit disk that has the same boundary values and integrable first-order distributional derivatives ?

We give the optimal geometric criterion for the interior Jordan domain so that there exists a Sobolev homeomorphic extension for any homeomorhic parametrization of the Jordan curve. The problem is partially motivated by trying to understand which boundary values can correspond to deformations of finite energy.

Title
Closed ideals and subideals associated to the algebra of compact-by-approximable operators.

Hans-Olav Tylli, Helsinki University.

Abstract

I will describe recent work [1] with Henrik Wirzenius (Tampere), where we construct explicit examples of non-trivial closed ideals $\mathcal{A}(X) \subset \mathcal{J} \subset \mathcal{K}(X)$ for various Banach spaces X that fail to have the approximation property. Here $\mathcal{K}(X)$ is the Banach algebra of compact operators on X, and $\mathcal{A}(X) := \overline{\mathcal{F}(X)}$ is the uniform closure of the bounded finite rank operators $\mathcal{F}(X)$. Our work was motivated by longstanding problems about the structure of the quotient algebra $\mathcal{K}(X)/\mathcal{A}(X)$, which is a very elusive object.

Results include a Banach space Z together with an uncountable lattice $\{\mathcal{J}_\alpha : \alpha \in \Lambda\}$ of closed ideals of $\mathcal{K}(Z)$, which are not ideals of the algebra $\mathcal{L}(Z)$ of bounded operators. This family has the property [2] that $\mathcal{J}_\alpha$ and $\mathcal{J}_\beta$ are isomorphic as Banach algebras whenever $\alpha \neq \beta$, which is not possible for closed ideals of $\mathcal{L}(Z)$. It is also shown that classical Banach spaces X, including $X = L^p$ for $p \neq 2$ and $X = C(0,1)$, admit a variety of non-trivial closed ideals $\mathcal{K}(X) \subset \mathcal{J} \subset \mathcal{S}(X)$ that are not ideals of $\mathcal{L}(X)$. Here $\mathcal{S}(X)$ is the closed ideal of $\mathcal{L}(X)$ consisting of the strictly singular operators.

REFERENCES

[1] Hans-Olav Tylli & Henrik Wirzenius : *Closed ideals in the algebra of compact-by-approximable operators*, J. Funct. Anal. 282 (2022), Paper No. 109328
[2] Hans-Olav Tylli & Henrik Wirzenius : *Exotic closed subideals of algebras of bounded operators*, Proc. Amer. Math. Soc. 152 (2024), 987–1002.

Title
Conformal welding and generalizations.

Steffen Rohde, University of Washington.

Abstract

We will discuss some equivalence relations on the sphere that arise naturally in complex dynamics and in probability theory. A classical theorem of Moore gives a condition when the quotient of the sphere by such an equivalence relation is again a topological sphere. We will consider the question when the quotient can be given a natural complex structure, give partial answers to this generalized conformal welding problem, and discuss some open problems.